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# TRANSIENT CONJUGATED HEAT TRANSFER BETWEEN A COOLING COIL AND ITS SURROUNDING ENCLOSURE

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**Abstract**—The transient heat transfer phenomena are investigated for the thermal entrance region of laminar pipe flows resulting from a step change in the entrance temperature, when coupled with the unsteady temperature variations in the surrounding enclosure, representing a refrigerator cabinet. The conventional Nusselt number is shown to be inadequate in the present study. The unsteady axial variations of a modified Nusselt number, bulk fluid temperature, and pipe wall temperature are presented over wide ranges of the parameters involved. The decay of the cabinet temperature is examined in great detail.

## NOMENCLATURE

| | |
|---|---|
| $A_c$ | cabinet surface area for heat transfer from ambient |
| $c_v$ | specific heat for the fluid in the cabinet |
| $D$ | pipe diameter |
| $h$ | local heat transfer coefficient inside the pipe, equation (13) |
| $h_e$ | modified local heat transfer coefficient inside the pipe, equation (15) |
| $k$ | thermal conductivity of the fluid in the pipe |
| $L$ | length of the pipe enclosed by the cabinet |
| $m$ | total mass of the fluid in the cabinet |
| $n$ | index for dimensionless time step |
| $Nu$ | conventional local Nusselt number, $h(2R)/k$ |
| $Nu_e$ | modified local Nusselt number, $h_e(2R)/k$ |
| $Nu_o$ | outside Nusselt number, equation (9) |
| $Pe$ | Peclet number, $u_m(2R)/\alpha$ |
| $q''_w$ | heat flux at the pipe wall |
| $Q''_w$ | dimensionless heat flux at the pipe wall, equation (18) |
| $r$ | radial coordinate |
| $R$ | pipe radius |
| $t$ | time |
| $T$ | temperature |
| $T_b$ | bulk temperature for the fluid in the pipe |
| $T_c$ | temperature of the fluid in the cabinet |
| $T_0$ | ambient temperature |
| $T_w$ | pipe wall temperature |
| $u$ | axial velocity |
| $u_m$ | mean axial velocity |
| $U_1$ | outside heat transfer coefficient, equation (3e) |
| $U_2$ | outside heat transfer coefficient in the ambient side, equation (2) |
| $x$ | axial coordinate |

Greek symbols

| | |
|---|---|
| $\alpha$ | thermal diffusivity of the fluid in the pipe |
| $\Delta\tau$ | dimensionless time step |
| $\eta$ | dimensionless radial coordinate |
| $\theta$ | dimensionless temperature difference |
| $\theta_a$ | dimensionless ambient temperature difference, equation (7) |
| $\theta_c$ | dimensionless cabinet temperature difference, equation (4) |
| $\lambda_1$ | parameter, equation (7) |
| $\lambda_2$ | parameter, equation (7) |
| $\xi$ | dimensionless axial coordinate |
| $\xi_L$ | dimensionless pipe length, equation (7) |
| $\tau$ | dimensionless time |

Subscripts

| | |
|---|---|
| b | bulk |
| c | cabinet |
| e | entrance |
| i | initial |
| w | wall |

## INTRODUCTION

AN ACCURATE prediction of the unsteady heat transfer phenomena has received greater attention recently because of its importance in the design of engineering systems involving heat exchanging devices. There are several studies on the transient heat transfer in channel flows in the literature, these were recently in ref. [1].

Through the use of the simple, efficient finite difference numerical schemes, Lin *et al.* [1, 2] successfully solved the transient two-dimensional energy equation for various flow conditions. The first paper examined the unsteady thermal entrance heat transfer in laminar pipe flows subject to a step change in ambient temperature. The transient thermal entrance laminar pipe flow heat transfer resulting from a step change in both the pressure gradient in the flow and entrance temperature was studied in the second paper.

The purpose of this paper is to investigate the transient thermal interaction between the cooling channel and the associated cabinet. Figure 1 shows a cooling fluid flowing through a pipe enclosed in a cabinet which is to be temperature controlled. Initially, the fluid in the cabinet and the fluid in the downstream portion of the circular channel ($x > 0$) are at the same uniform temperature $T_i$, while the fluid in the upstream portion ($x \leqslant 0$) is at a different uniform temperature $T_e$. The fluid in the channel is made to circulate at time

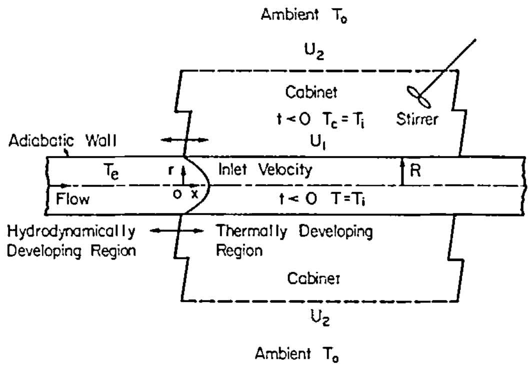


FIG. 1. Schematic diagram of the problem.

$t = 0$ by means of a pump, and heat exchange between the cabinet and the channel and between the ambient and the cabinet is initiated.

## ANALYSIS

The unsteady flow motion and energy transport in the cabinet, induced by the transient variation of the channel wall temperature, are very complicated. The rigorous analysis of the heat transport in the cabinet is beyond the scope of the present study. In order to overcome this complexity, a lumped-system analysis is employed to treat the transient heat transfer for the fluid in the cabinet [3]. More attention is paid to the transient heat transfer phenomena in the channel.

In the present study the following major simplifying assumptions are made:

(1) The ambient is isothermal ($T_0$ = constant).

(2) The fluid properties are temperature independent.

(3) The flow field is laminar and hydrodynamically fully developed.

(4) The overall heat transfer coefficients between the fluid in the pipe and the cabinet $U_1$ and between the ambient and the cabinet $U_2$ are constant and uniform.

(5) Viscous dissipation and free convection for the fluid in the channel are negligible.

(6) The axial heat conduction in the fluid can be neglected.

(7) The heat conductions in the pipe wall and cabinet wall have little effect.

(8) The temperature of the fluid in the cabinet is uniform in space but varies with time.

Resulting from assumptions 2 and 3, the velocity field for the fluid in the channel can be represented by Poiseuille's profile. The transient heat transport processes for the fluid in the channel can be described by

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}=\alpha\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \quad (1)$$

where $\alpha$ is the thermal diffusivity of the fluid in the pipe and $u = 2u_m(1-r^2/R^2)$. Based on the lumped-system analysis, the energy equation for the fluid in the cabinet is

$$mc_v\frac{dT_c}{dt}=U_2A_c(T_0-T_c)-\int_0^L U_1(T_c-T_w)\pi D\,dx, \quad (2)$$

where $m$ and $c_v$ are the mass and the specific heat of the material enclosed in the cabinet. $A_c$ is the cabinet surface area for heat transfer from the ambient and $T_w$ is the channel wall temperature. Equations (1) and (2) are subjected to the following initial and boundary conditions

$$t=0,\quad T=T_i, \quad (3a)$$

$$T_c=T_i, \quad (3b)$$

$$x=0,\quad T=T_e, \quad (3c)$$

$$r=0,\quad \frac{\partial T}{\partial r}=0, \quad (3d)$$

$$r=R,\quad -k\frac{\partial T}{\partial r}=U_1(T-T_c), \quad (3e)$$

where $U_1$ is the heat transfer coefficient based on the resistance of the channel wall and cabinet-side surface resistance.

Introducing the dimensionless variables

$$\begin{aligned}\xi &\equiv x/(R\,Pe),\\ \eta &\equiv r/R,\\ \tau &\equiv \alpha t/R^2,\\ \theta &\equiv (T-T_e)/(T_i-T_e),\\ \theta_c &\equiv (T_c-T_e)/(T_i-T_e).\end{aligned} \quad (4)$$

Equations (1)–(3) then become

$$\frac{\partial\theta}{\partial\tau}+(1-\eta^2)\frac{\partial\theta}{\partial\xi}=\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right), \quad (5)$$

$$\frac{d\theta_c}{d\tau}=\lambda_1(\theta_a-\theta_c)-\lambda_2\int_0^{\xi_L}(\theta_c-\theta_w)\,d\xi, \quad (6)$$

where

$$\begin{aligned}\lambda_1 &= \frac{U_2A_cR^2}{mc_v\alpha},\\ \lambda_2 &= \frac{2\pi U_1R^4\,Pe}{mc_v\alpha},\\ \theta_a &= (T_0-T_e)/(T_i-T_e),\\ \xi_L &= L/(R\,Pe),\end{aligned} \quad (7)$$

and

$$\tau=0,\quad \theta=1, \quad (8a)$$

$$\theta_c=1, \quad (8b)$$

$$\xi=0,\quad \theta=0, \quad (8c)$$

$$\eta=0,\quad \frac{\partial\theta}{\partial\eta}=0, \quad (8d)$$

$$\eta=1,\quad \frac{\partial\theta}{\partial\eta}+Nu_o\theta=Nu_o\theta_c, \quad (8e)$$

where the outside Nusselt number is defined in an unusual manner

$$Nu_o \equiv U_1 R/k, \tag{9}$$

because $k$ is the thermal conductivity of the fluid inside the pipe. In ref. [4], however, $Nu_o$ was designated as Biot number.

It is important to note that equations (5) and (6) are coupled through $\theta_w$ of equation (6) and the boundary conditions, equation (8). The major non-dimensional parameters governing the problem are $Nu_o$, $\lambda_1$, $\lambda_2$, and $\xi_L$.

## SOLUTION METHODOLOGY

The energy equation (5) describing the transient energy transport for the fluid in the channel is a partial differential equation, which is coupled with the integral–differential energy equation (6) for the fluid in the cabinet. In view of the impossibility of obtaining an analytic solution, the problem defined by the foregoing equations was solved numerically. Equation (5) is a parabolic type of PDE both in time and space, hence the solution can be marched from upstream to downstream for each time step. The Euler's predictor–corrector method is used to solve equation (6) with the integral term on the RHS expressed by the Trapezoidal Rule. Since equations (5) and (6) are coupled, they must be solved simultaneously. For each time step, equation (5) is solved first, using $\theta_c$ from the previous time step. With this $\theta$ distribution, $\theta_c$ is obtained by Euler's predictor method. Using the updated $\theta_c$, equation (5) is solved again and then $\theta_c$ is corrected by Euler's corrector method. The procedure can be repeated until the solution converges.

Employing backward difference for the unsteady energy term, upwind difference for the axial convection term [5], and central difference for the radial diffusion term, equation (5) is transformed into finite difference form, which is similar to those given in ref. [1]. The predictor solution of equation (6) can be obtained by

$$\theta^n_{c,p} = \theta^{n-1}_c + f(\theta^{n-1}_c, \theta_w)\Delta\tau_{n-1}, \tag{10}$$

where

$$f(\theta^{n-1}_c, \theta_w) = \lambda_1(\theta_a - \theta^{n-1}_c) - \lambda_2 \int_0^{\xi_L} (\theta^{n-1}_c - \theta_w)\,d\xi, \tag{11}$$

and the integral is evaluated by the Trapezoidal Rule. This solution is then corrected by

$$\theta^n_{c,c} = \theta^{n-1}_c + 0.5[f(\theta^{n-1}_c, \theta_w) + f(\theta^n_{c,p}, \theta_w)]\Delta\tau_{n-1}. \tag{12}$$

In the above calculation, the updated $\theta_w$ is always used.

Because the drastic temperature transients only occur in the region close to the channel entrance and only for a short time, highly non-uniform axial grid and time step are employed. But a uniform grid is used in the radial direction. A total of 51 and 41 grid lines are placed in the axial and radial directions, respectively, and 151 time steps are used to cover the time duration for the temperature transient to disappear. It takes less than 300 s to solve the problem with a CDC (control data corporation) 6600 computer system.

## RESULTS AND DISCUSSION

With the transient temperature distribution for the fluid in the channel obtained by the procedures outlined above, the major parameters of interest can be evaluated. The conventional local heat transfer coefficient inside the pipe is normally defined as

$$h \equiv \frac{q''_w}{T_b - T_w} \equiv \frac{-(\partial T/\partial r)_w}{(T_b - T_w)/k}. \tag{13}$$

By making use of the boundary condition in equation (3e), the conventional local Nusselt number inside the pipe can be expressed as

$$Nu \equiv \frac{h(2R)}{k} \equiv \frac{2Nu_o(\theta_w - \theta_c)}{\theta_b - \theta_w}, \tag{14}$$

where $\theta_b$ and $\theta_w$ are the dimensionless bulk fluid temperature and channel wall temperature, respectively.

As was shown in ref. [6], the conventional Nusselt number cannot be used to interpret the physical situation in certain problems, which will also become evident in this study. A modified heat transfer coefficient based on the difference of the initial temperature $T_i$ and the entrance temperature $T_e$ is introduced

$$h_e \equiv \frac{q''_w}{T_i - T_e} \equiv \frac{-k(\partial T/\partial r)_w}{T_i - T_e}, \tag{15}$$

and the modified Nusselt number becomes

$$Nu_e \equiv \frac{h_e(2R)}{k} = 2Nu_o(\theta_w - \theta_c). \tag{16}$$

Comparison of equation (14) with equation (16) shows

$$Nu_e = Nu(\theta_b - \theta_w). \tag{17}$$

The dimensionless heat flux at the channel wall can be evaluated by

$$Q''_w \equiv \frac{q''_w}{(k/D)(T_i - T_e)} = Nu_e. \tag{18}$$

Therefore, $Nu_e$ is a real indication of the dimensionless channel wall heat transfer rate. The actual wall heat transfer rate is

$$q''_w = Nu_e\left[\frac{k}{D}(T_i - T_e)\right]. \tag{19}$$

In all the numerical calculations, $\theta_a$ is assumed to be unity, i.e. $T_i = T_0$. The transient heat transfer phenomena for the fluid in the channel and for the fluid in the cabinet are investigated over wide ranges of the four major parameters, $Nu_o$, $\lambda_1$, $\lambda_2$, and $\xi_L$. The steady-state temperature distributions for the fluid in the channel for various values of $Nu_o$ were found to be in excellent agreement with those obtained by Hsu [4].

For the convenience of discussion, $T_c$ is assumed to be lower than $T_i$. Normally the fluid in the channel will receive heat from the fluid in the cabinet as the cabinet is cooled down. The axial variations of the conventional Nusselt number at various instants of time are shown in Fig. 2 for $Nu_o = 2, \lambda_1 = 0.25, \lambda_2 = 5$, and $\xi_L = 0.5$. For given $\tau$, $Nu$ changes from positive to negative at a certain axial location and changes back to the positive values at another location. This could have indicated that heat exchange between the cabinet and the channel changes direction twice, occurring when $Nu$ changes sign. In fact, the direction of the channel wall heat transfer only changes once, as evident from the bulk fluid temperature and wall temperature distributions in Fig. 3. This clearly indicates the inappropriate definition of the conventional Nusselt number in the present problem. The presentation of the results through the use of the modified Nusselt number as given in Fig. 4 avoid this confusion. Moreover, the magnitude of $Nu_c$ can actually represent the magnitude of wall heat transfer rate [6]. For given $\tau$, $Nu_c$ is negative because $\theta_w < \theta_c$ except in the far downstream region where $\theta_w > \theta_c$ and $Nu_c$ is positive. Therefore, the fluid in the channel receives heat from the cabinet in the near entrance region, but rejects heat to the cabinet in the far downstream region. It can also be seen, at a given axial location, that the wall heat transfer rate increases with time at first, but it decreases with time after a certain period of time. Because of the inappropriateness of the conventional Nusselt number in the present study, only the results for the modified Nusselt number will be given hereafter.

The influence of the outside Nusselt number on the transient heat transfer are shown in Figs. 5 and 6 for $Nu_o = 0.5$. Comparing the corresponding curves with

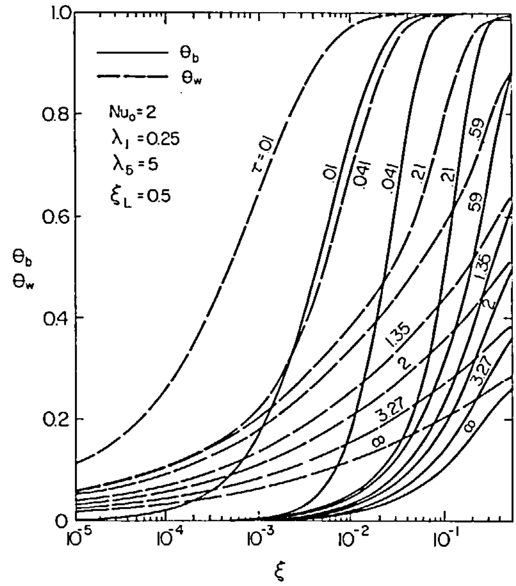

FIG. 3. Unsteady bulk fluid temperature and wall temperature distributions for $Nu_o = 2$.

those for $Nu_o = 2$, it is found that the larger the outside Nusselt number, the larger the modified Nusselt number and the bulk fluid temperature. This simply results from the fact that the increase in $Nu_o$ can enhance the radial energy diffusion for the fluid in the channel. It is important to note that the system does not approach steady state quicker for larger $Nu_o$, which is found for the transient resulting from a simple step change [1, 2]. This can be attributed to the fact that both the fluid in the channel and the fluid in the cabinet are undergoing transient changes.

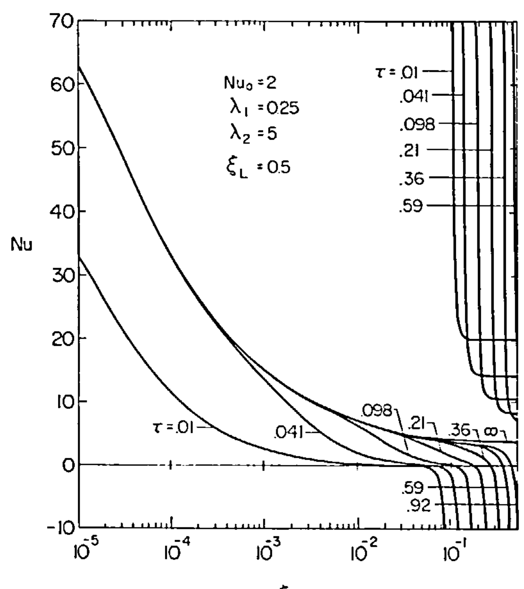

FIG. 2. Transient axial variations of conventional Nusselt number for $Nu_o = 2$.

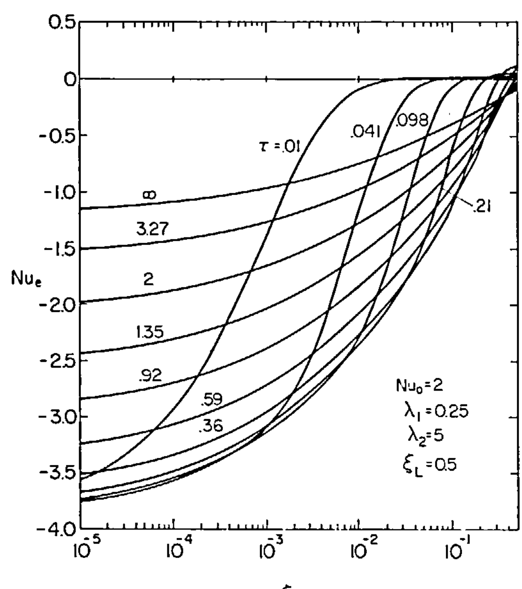

FIG. 4. Transient axial variations of modified Nusselt number for $Nu_o = 2$.

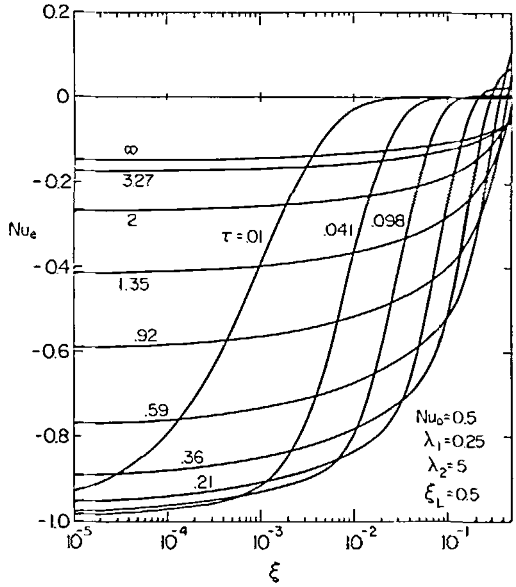


FIG. 5. Transient axial variations of modified Nusselt number for $Nu_o = 0.5$.

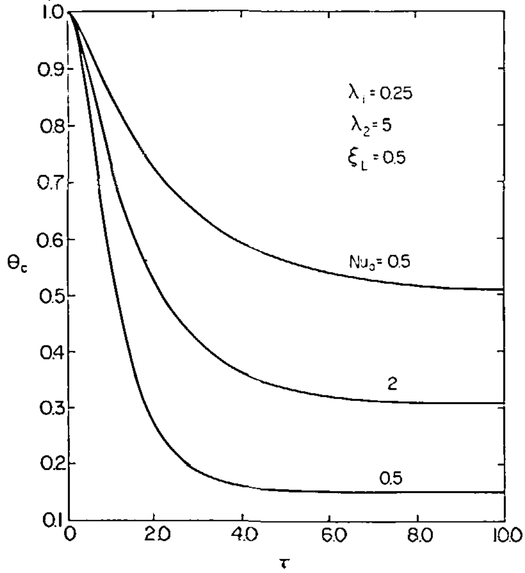


FIG. 7. The influence of outside Nusselt number on the transient variations of cabinet temperature.

The time variations of the cabinet temperature for various outside Nusselt numbers are shown in Fig. 7. Quite surprisingly, the larger outside Nusselt number results in a slower rate of cabinet cooling and a higher asymptotic cabinet temperature. This is somewhat beyond our expectation that larger $Nu_o$ results in a higher wall heat transfer rate, which in turn will cause larger cabinet cooling. Examining the expressions for the parameters $Nu_o$, $\lambda_1$, $\lambda_2$, and $\xi_L$ in equation (7), it is observed that they are not independent of each other. Thus, a change in $Nu_o$ normally causes changes in the other parameters. Therefore, the hypothetical situation of varying $Nu_o$ is used here only for reasons of discussion.

Considering a more practical situation, the heat transfer coefficient $U_1$ is increased by 2.5 times and decreased by 4 times, so are $Nu_o$ and $\lambda_2$, and other parameters remain constant. Figure 8 shows the decay of the cabinet temperature with time for these changes. It is distinctly seen that the larger $U_1$ results in a faster decay in cabinet temperature and a lower asymptotic cabinet temperature. This is physically reasonable.

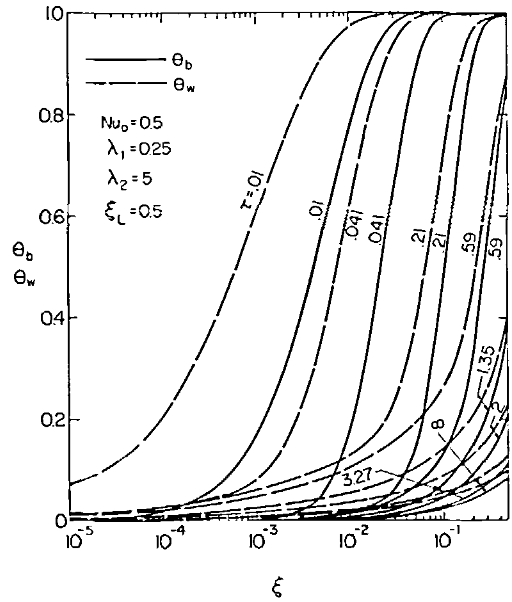


FIG. 6. Transient axial variations of bulk fluid and wall temperature for $Nu_o = 0.5$.

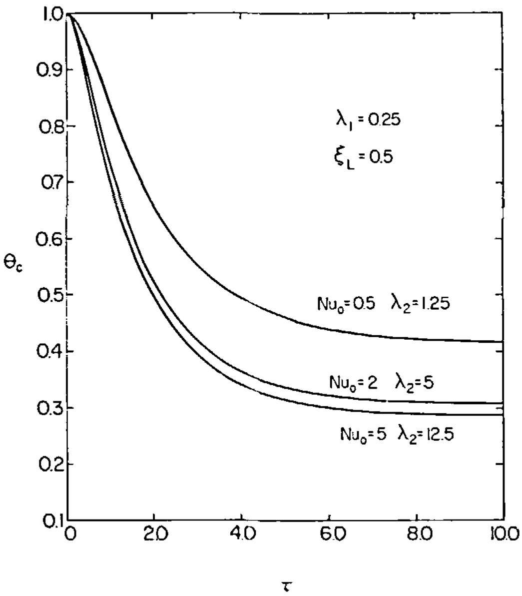


FIG. 8. The influence of the outside heat transfer coefficient on the unsteady variations of cabinet temperature.

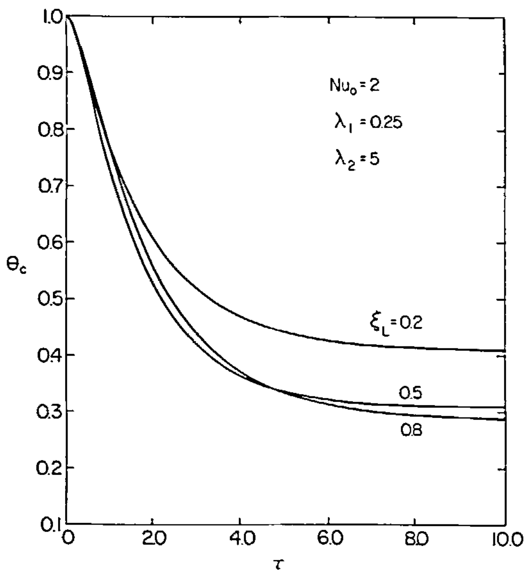


FIG. 9. The influence of channel length on the transient variations of cabinet temperature.

The effects of the cooling channel length on the transient variation of the cabinet temperature are presented in Fig. 9. It is expected that longer channel length, allowing more surface area for heat transfer, will result in a lower steady-state cabinet temperature. This is confirmed by the results in Fig. 9. It is worth noting that the increase of $\xi_L$ from 0.5 to 0.8 has only a slight influence on the steady cabinet temperature. This is believed to be due to the fact that heat exchange between the channel and the cabinet is rather ineffective in the far downstream portion as seen in the $Nu_\varepsilon$ plot, Fig. 4. Also, it is important to note that the cabinet cooling rate is not faster for larger $\xi_L$ in the initial period. This could be attributed to the simultaneous transient temperature variations in the channel and cabinet and heat exchange between them.

It is also found in the numerical calculation that a larger $\lambda_2$ results in faster decay in cabinet temperature and lower steady cabinet temperature. For constant $Nu_o$ and $\xi_L$, the steady-state temperature of the cabinet is the same as long as the ratio of $\lambda_1$ and $\lambda_2$ is the same. When the unsteady terms in equations (5) and (6) are neglected, $\theta_c$ should be only dependent on $\lambda_1/\lambda_2$ for constant $Nu_o$ and $\xi_L$.

Finally, it needs to be mentioned that the transient variations of $Nu_\varepsilon$ are mainly caused by the change in $Nu_o$. The other parameters, $\lambda_1$, $\lambda_2$, and $\xi_L$, have only slight influences.

## CONCLUDING REMARKS

The transient heat transfer phenomena in the thermal entrance region of laminar pipe flow and the associated cabinet have been investigated over wide ranges of the governing parameters, $Nu_o$, $\lambda_1$, $\lambda_2$, and $\xi_L$. The unsteady axial variations of the modified Nusselt number, bulk fluid temperature, and wall temperature for the fluid in the channel are presented and discussed in great detail. The decay of the cabinet temperature is also examined. The major findings can be summarized as follows:

(1) The unsteady local Nusselt number for the fluid in the channel deviates considerably from its corresponding steady value.

(2) The increase in the outside Nusselt number results in a higher heat exchange between the channel and the cabinet.

(3) The heat exchange between the channel and the cabinet is rather ineffective in the downstream region. Thus, the increase of the channel length over a certain critical value has negligible effect on the cooling of the cabinet.

(4) For given $Nu_o$ and $\xi_L$, the final state that the cabinet will attain depends only upon the ratio $\lambda_1/\lambda_2$.

(5) The transient axial variations of $Nu$ is mainly governed by the outside Nusselt number, so is the channel wall heat transfer rate.

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## TRANSFERT THERMIQUE VARIABLE CONJUGUE ENTRE UN SERPENTIN REFRIGERANT ET LA CAVITE QUI L'ENTOURE


Résumé—Des phénomènes de transfert thermique variables sont étudiés pour la région d'entrée thermique d'un écoulement laminaire en tube, avec changement de température en échelon à l'entrée et couplage avec les variations de température dans l'enceinte qui représente une chambre de réfrigérateur. Le nombre de Nusselt ne convient pas dans cette étude. Les variations axiales instationnaires d'un nombre de Nusselt modifié, de la température de mélange du fluide et de la température de la paroi sont présentées dans de larges domaines des paramètres. La décroissance de la température de la chambre est examinée en détail.

## INSTATIONÄRE KONJUGIERTE WÄRMEÜBERTRAGUNG ZWISCHEN EINER KÜHLSCHLANGE UND IHREM UMGEBENDEN RAUM

**Zusammenfassung**—Das instationäre Wärmeübertragungsverhalten wird im thermischen Eintrittsgebiet für laminare Rohrströmung bei sprungartiger Änderung der Eintrittstemperatur untersucht, wobei eine Kopplung an die instationären Temperaturänderungen des umgebenden Raumes, der das Innere eines Kühlschranks darstellt, vorliegt. Es wird gezeigt, daß die konventionelle Nusselt-Zahl im vorliegenden Fall ungeeignet ist. Der instationäre axiale Verlauf einer modifizierten Nusselt-Zahl, die mittlere Flüssigkeitstemperatur und die Rohrwandtemperatur werden über einen weiten Bereich der beteiligten Parameter dargestellt. Der Temperaturabfall im Kühlschrank wird dabei sehr detailliert untersucht.

## НЕСТАЦИОНАРНЫЙ СОПРЯЖЕННЫЙ ТЕПЛОПЕРЕНОС МЕЖДУ ОХЛАЖДАЮЩИМ ЗМЕЕВИКОМ И ОКРУЖАЮЩИМ ПРОСТРАНСТВОМ

**Аннотация**—Исследован нестационарный теплообмен на термическом начальном участке при ламинарном течении в трубе, ступенчатом изменении температуры на входе и нестационарном температурном режиме в объеме холодильной камеры. Для условий данного исследования показана неадекватность обычно используемого числа Нуссельта. Нестационарные изменения по оси модифицированного числа Нуссельта, температуры жидкости в объеме и температуры стенки трубы представлены в широких диапазонах исследуемых параметров. Подробно изучен процесс понижения температуры холодильной камеры.